

Decoherence, Classical Properties and Entanglement of Quantum Systems

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We discuss the properties of decoherence and its role in the appearance of classical properties in open quantum systems. In particular, it is used for classification of pure states with respect to their ability to persist despite the environmental monitoring.

Key words: Decoherence; Measurement Problem; “Classical” States; Entanglement.

Quantum mechanics, whose basic laws were formulated in the twenties, still remains the most fundamental theory we know. Although it was originally conceived as a theory of atoms, it has shown a wide range of applicability, making it more and more evident that the formalism describes some general properties of Nature. However, despite its successes, there is still no consensus about its interpretation, with the main questions being centered on quantum measurements. At the end of the century, quantum mechanics remains both intact and puzzling. Since the dynamics of quantum mechanics described by the Schrödinger equation is linear, it follows that, after an interaction between a quantum system and a measuring apparatus, the state of the composite system is not, in general, an eigenstate of the measured observable, and not an eigenstate of the observable functioning as a pointer. Hence, neither the measured observable nor the pointer reading have determined values. Therefore, measurement-like processes would necessarily produce such non-classical states as in the infamous example of Schrödinger’s cat. A superposition of being dead and alive should result in an entirely new state, in the same sense as the superposition of a K meson and its antiparticle does.

The main reason for this annoying situation seems to be based on the assumption that it is possible to isolate systems from their environment. When we drop it

as unjustified and consider quantum systems as open ones, we obtain, due to the decoherence process, a new perspective for understanding the emergence of classical properties.

In recent years decoherence has been widely discussed and accepted as the mechanism responsible for the appearance of classicality in quantum measurements and the absence, in the real world, of Schrödinger-cat-like states [1 - 4]. The basic idea behind it is that classicality is an emergent property induced in quantum open systems by their environment. It is marked by the dynamical transition of the vast majority of pure states of the system to statistical mixtures. In other words, decoherence is a process of a continuous interaction between the system and its environment which results in limiting the validity of the superposition principle in the Hilbert space of the system. This resolves the measurement problem essentially in the following way: Since any realistic measuring apparatus is macroscopic it necessarily interacts with its environment, for example with the gravitational force, and so, almost instantaneously after a measurement, the reduced state of the measuring instrument is, for all practical purposes, indistinguishable from a state representing a classical probability distribution over determined but unknown values of the pointer observable. The information required to exhibit quantum interference effects between distinct

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pointer states is immediately lost in the external degrees of freedom.

In consequence, it leads to the appearance of environment-induced superselection rules which precludes all but a particular subset of states from stable existence. A thorough analysis of the structure of environment-induced superselection rules has been presented in [5, 6]. It is worth noting that, in general, not only superpositions $\lambda|v\rangle + \lambda'|w\rangle$, $|\lambda|^2 + |\lambda'|^2 = 1$, of vectors from different coherent subspaces are empirically indistinguishable from mixtures $|\lambda|^2 P_v + |\lambda'|^2 P_w$, where $P_v = |v\rangle\langle v|$ and $P_w = |w\rangle\langle w|$. It may happen that some non-proportional vectors from one coherent space still determine the same quantum state. The set of such vectors is then represented by an r -dimensional sphere, so-called the generalized ray [7], r being the dimension of an irreducible subspace of commuting physical observables. It follows then that the minimal stable projection has the dimension r . Therefore, in such a case superselection operators form a noncommutative algebra, whereas in the measurement problem it was necessarily commutative. If $r = 1$, then we recover the scheme for the measurement processes.

On the other hand, decoherence singles out a subset of preferred states which behave in an effectively classical, predictable manner. Such states are distinguished by their ability to persist despite the environmental monitoring, and therefore they are the ones in which the quantum system may be observed. They are called “classical”. Let us discuss this notion in a more detailed way. We restrict our considerations to the case when the interaction leads to a Markovian master equation, that is the evolution of any state of the system is given by a quantum Markov semigroup. Let us recall that the quantum Markov semigroup is a strongly continuous semigroup of completely positive, contractive and trace preserving superoperators. At first, let us comment on crucial differences between states of classical and quantum systems. One of the most characteristic features distinguishing classical from quantum states is their sensitivity to measurements. In classical physics we could perform many kinds of measurements which would not disturb the system in an essential way. A measurement can increase our knowledge of the state of the system but, at least in principle, it has no effect on the system itself. By contrast, in quantum mechanics it is impossible to find out what the state is without, at the same time, changing it in a way determined by the

measurement. According to the von Neumann projection postulate the outcome will be, in general, represented by a density matrix. Therefore, as a convenient measure of the influence of the environment on the state, we take the measure of the loss of its purity expressed in terms of the linear entropy $S_{\text{lin}}(\rho) = \text{tr}(\rho - \rho^2)$ [8].

Let \mathcal{S} denote the set of all states of the quantum system. By a state we always mean a pure state, whereas for a mixed state we reserve such notions like density matrix or statistical state. Hence \mathcal{S} consists of unit vectors from a Hilbert space \mathcal{H} determined up to a phase factor $[|\psi\rangle]$ or, in other words, of one-dimensional projectors in \mathcal{H} . Hence $[|\psi\rangle]$ is the abstract class of unit vectors with respect to the following equivalence relation: $|\psi\rangle \equiv |\psi'\rangle$ if $|\psi\rangle = e^{i\alpha}|\psi'\rangle$ for some $\alpha \in \mathbf{R}$. Let us notice that the scalar product of two distinct states is not well defined but its absolute value is. Also the one-dimensional projector $e_\psi = |\psi\rangle\langle\psi|$ does not depend on the choice of a state vector $|\psi\rangle$. A subset of robust (completely stable) states \mathcal{S}_0 is defined as

$$\mathcal{S}_0 = \{|\psi\rangle \in \mathcal{S} : S_{\text{lin}}(T_t e_\psi) = 0 \forall t \geq 0\}$$

Obviously, any state from \mathcal{S}_0 will remain pure during the evolution and so remain in \mathcal{S}_0 . Therefore, elements from \mathcal{S}_0 are the most probable candidates for “classical” states. However, perfect predictability alone does not suffice to accomplish our goal. For example, if the interaction was weak, then all states would evolve in a unitary way yielding $\mathcal{S}_0 = \mathcal{S}$. But all states cannot be classical. Therefore, another feature distinguishing quantum from classical states, namely the validity of the superposition principle, has to be taken into account. In quantum mechanics this principle guarantees that any superposition of two distinct, and not necessarily orthogonal states is again a legitimate quantum state. By contrast, classical states do not combine into another state. The only situation when their combination can be considered is inevitably tied to probability distributions on the phase space. Taking this into account, we further specified “classical” states as those whose all non-trivial superpositions deteriorate to the stable density matrices, and so, after some decoherence time, are practically indistinguishable from classical probability distributions. In other words, non-trivial superpositions of classical states cannot be robust. We can introduce now the following definition.

Definition. A state $|\psi\rangle \in \mathcal{S}$ is called “classical” if $|\psi\rangle \in \mathcal{S}_0$ and for any $|\phi\rangle \in \mathcal{S}_0$, $[|\phi\rangle] \neq [|\psi\rangle]$, $S(|\psi\rangle, |\phi\rangle) \cap \mathcal{S}_0 = \emptyset$, where $S(|\psi\rangle, |\phi\rangle)$ denotes the collection of all states being non-trivial superpositions of $|\psi\rangle$ and $|\phi\rangle$. The collection of all “classical” states is denoted by \mathcal{S}_c .

Under the technical assumption that the semigroup T_t does not also increase the biggest eigenvalue of a statistical state, we can fully characterize the structure of the set \mathcal{S}_c .

Theorem [9]. If $\mathcal{S}_c \neq \emptyset$, then it consists of a family, possibly finite, of pairwise orthogonal states $\{\psi_1, \psi_2, \dots\}$ such that $T_t(e_{\psi_i}) = e_{\psi_i}$ for all $t \geq 0$ and any index i .

Therefore, it turned out that “classical” states can form only a discrete set of pairwise orthogonal states, so-called pointer basis.

However, it should be noted that they may not exist at all. In such a case the predictability sieve was introduced [8, 10, 11]. It is a procedure which systematically explores states of an open quantum system in order to arrange them and next put on a list, starting with the most predictable ones and ending with those, which are most affected by the environment. For that purpose we introduce a quantity

$$\lambda(e_\psi) = \frac{1}{2} \frac{d}{dt} S_{\text{lin}}(T_t e_\psi)|_{t=0},$$

which measures the production rate of the linear entropy for small times. Clearly, the states having the minimal value of λ , and so being on the top of the list, can be thought of as the most classical ones. By combining predictability with the previously introduced principle expressing the fact that any superposition of two distinct preferred states cannot belong to the same class of stability, the notion of “classical” states may be also introduced in such a more general case. In a number of examples [9 - 12] it was demonstrated that such states are just coherent states associated with some symmetry group. They are those quantum states which correspond to points in a classical phase space. It is worth noting that in both cases, when there exist robust states or states which offer the minimal production rate of linear entropy, the induced classical properties depend on the type of interaction between the system and its environment, that is on the dissipative part in the master equation derived for the reduced density matrices.

On the other hand we can look at the other end of the list of states arranged according to the degree how fast they deteriorate to density matrices. It turned out that for a quantum Markov semigroup for a bipartite system associated to a “fuzzy” measurement of a continuous family of non-commuting observables of the component subsystems, the most unstable states are maximally entangled while the most stable ones are just separable states. More precisely, suppose that the dissipative part in the Markov generator is of the form

$$L_D(\rho) = \kappa \left[\int_{\mathbf{CP}^{n-1}} d\mu(\mathbf{n}) (P_A(\mathbf{n}) \otimes I_B) \cdot \rho (P_A(\mathbf{n}) \otimes I_B) - \rho \right].$$

This is a straightforward generalization of the formula $\sum P_i \rho P_i - \rho$, to the case when there is an overcomplete family of n -dimensional projections of the form $P_A(\mathbf{n}) \otimes I_B$. For simplicity, we assumed here that both component systems are n -dimensional, that is they are represented by $n \times n$ matrices. \mathbf{CP}^{n-1} denotes the complex projective space for one subsystem, i. e. the space of all pure states. The measure $d\mu(\mathbf{n})$ is a unique $U(n)$ -invariant measure on \mathbf{CP}^{n-1} normalized in such a way that

$$\int_{\mathbf{CP}^{n-1}} d\mu(\mathbf{n}) (P_A(\mathbf{n})) = I_A$$

and $\kappa > 0$ is a coupling constant. Calculating the quantity λ for the semigroup T_t generated by the master equation

$$\dot{\rho} = -i[H, \rho] + L_D(\rho)$$

we obtain that

$$\lambda(e_\psi) = \frac{2}{3} \kappa \left(1 - \frac{1}{n} \text{tr}(\text{tr}_A e_\psi)^2 \right)$$

where tr_A denotes the partial trace with respect to subsystem A . It is not difficult to identify the states for which λ acquires the minimal and maximal possible value.

Theorem. $\lambda(e_\psi) = \lambda_{\min}$ if and only if $|\psi\rangle = |\psi_A\rangle |\psi_B\rangle$. $\lambda(e_\psi) = \lambda_{\max}$ if and only if $\text{tr}_A e_\psi = I_B/n$ and $\text{tr}_B e_\psi = I_A/n$.

In other words, λ takes its minimal value for separable states, and the maximal one for the most entangled

states. In the case of $n = 2$ any maximally entangled state coincides, up to a local unitary isomorphism, with the Schrödinger-cat state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

In such a state the measurement of the spin of the first particle along any axis leads to a definitive prediction concerning the spin of the second particle along the

same axis. What is more, the two spin particles described by that state cannot be regarded as possessing individual properties at all. Therefore, as it is widely accepted [13], entangled states are strongly affected by such a measurement-like interaction between the composite system and its environment. Moreover, as was shown in [14], the production rate of linear entropy in such a case may also serve as another measure of entanglement.

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